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and

$$r=\frac{3V}{a+b+c+d}.$$

Also

$$V = AO_3BC + O_3BCD + O_3ACD - O_3ABD = \frac{r_3}{3}(a+b-c+d)$$

and

$$r_3=\frac{3V}{a+b-c+d}.$$

Similarly,

$$r_1=\frac{3V}{(-a+b+c+d)},\quad r_2=\frac{3V}{(a-b+c+d)},\quad \text{and}\quad r_4=\frac{3V}{(a+b+c-d)}.$$

Therefore

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4}=\frac{(-a+b+c+d)+(a-b+c+d)+(a+b-c+d)+(a+b+c-d)}{3V}$$

$$=\frac{2a+2b+2c+2d}{3V}=\frac{2}{r}.$$

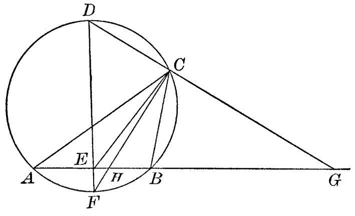
Also solved by A. M. Harding and Elmer Schuyler.

422. Proposed by J. SCHEFFER, Hagerstown, Md.

Construct strictly by use of elementary plane geometry, a triangle having given the product of two sides, the median to the third side, and the difference of the angles to the third side.

Solution by F. M. Morgan, Dartmouth College.

Suppose ABC is the required triangle circumscribed by a circle. Draw the internal and external bisectors of $\angle C$ meeting the circle in points F and D. Draw FD.



Since $\angle DCF$ is a right angle, FD is a diameter and bisects AB. Produce DC to meet AB in G. Since $\angle FEG = \angle FCG = 90°$, a circle will pass through points F, E, C, G. Therefore, $\angle EFH = \angle EGC = \frac{1}{2}$ of the difference of the base angles of triangle ABC. This angle is given and will be called α.

Now $\angle EHC = 90° + \alpha$ and is therefore known. Moreover, $CH \cdot CF = CA \cdot CB$ which is a given product.

Construction. Take line CE equal to the given median. Upon it describe the segments of two circles, one to contain an angle α, and the other an angle $90°+\alpha$. Draw line through C cutting these circles so that $CH \cdot CF = CA \cdot CB$. To do this produce CE to I so that $CE \cdot CI = CA \cdot CB$ and construct an angle at I equal to $90° + \alpha$. Then F is the point where the side of this angle meets the segment containing $\angle \alpha$; for E, H, F, I are on a circle and $CE \cdot CI = CH \cdot CF$.

Draw FE and let it meet the perpendicular to FC at C in point D. Pass a circle through D, C, F and where this circle meets EH extended will be points A and B. The triangle ABC is then the required triangle.

Also solved by A. H. HOLMES, DAVID F. KELLEY, ELMER SCHUYLER, and A. M. HARDING.

A solution of 416 was received from S. W. REAVES too late for notice in the November, 1913, issue.

No solution has been received for 421.

CALCULUS.

A solution of 334 has been received to which the author neglected to sign his name. Two solutions of 327, which was reprinted in the February, 1913, issue, p. 68, have been received. Also solutions are in hand for 336, 339, 341, and 343. All of these will appear in the next issue. Meanwhile we hope to hear from 337, 338, 340, 342, and 344, which include all published up to November, 1913.

MECHANICS.

No solutions have been received for the following problems in mechanics: 246 published in October, 1911; 266 in December, 1911; 268 in January and 269 in April, 1912, and none of those published in 1913, namely 271–283.

Solutions of these problems are desired.

Please remember that problems 276–279 are incorrectly numbered 271–274. See page 258 of the October, 1913, issue.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

190. Proposed by H. C. FEEMSTER.

Show that, if n is a prime number and r an integer less than n, then

$$(r-1)!\,(n-r)! + (-1)^{r-1} = M \cdot n,$$

where M is an integer.

SOLUTION BY THOMAS E. MASON, Indiana University.

From the Wilson theorem we have

$$(n-1)! + 1 \equiv 0 \mod n. \tag{1}$$

We have also,

$$\begin{aligned}(n-1)(n-2)\cdots(n-r+1) &\equiv (-1)(-2)\cdots(-r+1) \mod n.\\ &\equiv (-1)^{r-1}(r-1)! \mod n. \qquad (2)\end{aligned}$$

Making substitution from (2) in (1) we get

$$(-1)^{r-1}(r-1)!(n-r)! + 1 \equiv 0 \mod n,$$

or

$$(r-1)!(n-r)! + (-1)^{r-1} \equiv 0 \mod n,$$

which proves the theorem.